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CVA with wrong-way risk and correlation between defaults: An application to an interest rate swap

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Abstract: This paper presents a counterparty credit risk adjustment model to value over-the-counter financial derivatives. To do so, a bilateral credit valuation adjustment with wrong-way risk (WWR) and dependency between the defaults of both contract parties is developed in line with the Hull-White model (2012), which calculates default probabilities using a hazard rate modelled as an exponential function dependent on the value of the derivative. The model proposed incorporates a modified hazard rate for each entity, which includes the company's own exposure to credit risk attributable to the other entity's default. By so doing, a correlation between the respective defaults of the entities party to the financial derivative is added. The model developed is also applied to obtain the fair value of an interest rate swap and the results obtained, using Monte Carlo simulation, demonstrate that the value of this swap adjusted to the credit risk falls when the dependency between the entities' defaults is considered.

CÓDIGOS JEL:
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PALABRAS CLAVE:
Riesgo de crédito; Probabilidad de *default*; *Wrong-way risk*; Dependencia; *Swap*

Resumen: Este artículo presenta un modelo de ajuste del riesgo de crédito de contraparte para valorar derivados financieros *over-the-counter*. Para ello, se lleva a cabo un ajuste de la valoración del riesgo de crédito bilateral con *wrong-way risk* (WWR) y dependencia entre los defaults de ambas partes del contrato, en línea con el modelo de Hull-White (2012) que calcula las probabilidades de *default* utilizando una tasa de hazard modelizada como una función exponencial que depende del valor del derivado. El modelo propuesto incorpora una tasa de hazard modificada para cada entidad, que incluye la exposición al riesgo de crédito de la empresa atribuible al incumplimiento de la contraparte. De esta forma se añade la correlación entre los respectivos incumplimientos de las entidades que contratan el derivado financiero. El modelo desarrollado también se aplica para obtener el valor razonable de un *swap* de tipos de interés y los resultados obtenidos, utilizando el método de simulación de Monte Carlo, demuestran que el valor de este *swap* ajustado al riesgo de crédito disminuye cuando se considera la dependencia entre los incumplimientos de las entidades.

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1. Introduction: literature review

The aim of this paper is to develop a model that can calculate the counterparty credit risk adjustment for valuing over-the-counter (OTC) financial derivatives. This counterparty credit risk adjustment, referred to as a credit valuation adjustment (CVA), has been the object of study in the specialized financial literature in recent years, in particular since 2007, when the financial crisis highlighted the importance of a correct valuation of counterparty credit risk (Kao, 2016). Indeed, an accurate credit risk assessment is of extreme importance from an accounting point of view and is a regulatory capital quantification requirement under Basel III (Basel Committee on Banking Supervision, 2011).

The counterparty credit risk inherent to OTC financial derivatives is the risk that one of the counterparties to that financial contract defaults before the maturity date and, as a consequence, fails to meet all the payments to which it is contractually committed (Gregory, 2010). The CVA models developed over the last few years seek to value this counterparty risk and so fix the 'fair value' of the derivative asset by adjusting its risk-free value (assuming, that is, there is no default) with a credit risk or CVA provision. Depending on the nature of the financial derivative contract, the model will consider either only the counterparty credit risk, i.e. the unilateral CVA, or the credit risk of the entity that values the derivative asset, i.e. the bilateral CVA.

Broadly speaking, the basic CVA model is determined by three components: a dynamic discount function, the expected exposure to risk of the counterparty credit over time, and the probability of default. This second component quantifies expected losses should the counterparty default and, logically, is determined by the derivative's value at each moment in time.

In their origin, the CVA models did not contemplate any dependency between these components (see, in this line, Sorensen and Bollier, 1994, and Brigo and Masetti, 2005). However, later, so as to capture the credit risk adjustment more precisely, the dependency between default and value, on the one hand, and between default and the credit risk exposure of the derivative asset, on the other, were taken into consideration – referred to as the wrong-way risk (WWR) and which increases the value of the credit risk adjustment. Initially, Basel III recommended incorporating the WWR to calculations of the CVA by applying a multiplier to the expected exposure so as to obtain the default exposure. The value of this multiplier was fixed at 1.4 for all companies except banking institutions, for which a value of 1.2 was advised (see García Céspedes et al., 2010, for an analysis of the multiplier model).

An alternative way of introducing dependency between default and credit risk exposure is by using copulas, an approach that entails a change in risk measurement (see, in this regard, García Céspedes et al., 2010; Gregory, 2010; Rosen and Saunders, 2012; Cherubini, 2013; Böcker and Brunnbauer, 2014; and Pan and Khandrika, 2019).

A third way of introducing WWR involves adopting an empirical approach, which modifies the probability of default by specifying the hazard rates as functions of either risk exposure, the value of the derivative or, even, a variable affecting the former (see, in this regard, the models developed by Brigo and Pallavicini, 2007; Hull and White, 2012; Ruíz et al., 2013; and Gargouri et al., 2017).

The model presented in this paper adheres to this last approach. More specifically, it is based on the Hull-White (2012) model which, in order to introduce WWR, calculates default probabilities by means of a hazard rate modelled as an exponential function dependent on the value of the derivative or a portfolio of derivatives. As such, the model proposed herein can be considered an extension of the Hull-White model (2012) since it adds the dependency between the defaults of both parties to the contract. Specifically, a modified hazard rate is proposed that adds to the exponential function modelling this rate the company's own exposure to credit risk attributable to the other entity's default. By so doing, a correlation between the respective defaults of the entities party to the financial derivative is introduced, which should lead to an increase in the credit risk adjustment, a result that is borne out by the outcomes reported below.

Here, a plain vanilla interest rate swap (IRS) is chosen from among different OTC derivatives and its fair value is calculated. An IRS is a derivative asset in which two traders exchange periodic payments of interest, referenced to the same currency and calculated on a constant principal amount but with two different benchmark interest rates: one fixed and the other floating without any margin being added to the benchmark rate. Previous studies that have assessed the CVA for an IRS include Cerný and Witzany (2015), Smith (2015), Kriváncková and Zlatosová (2017), Ben-Abdallah et al. (2019) and Badía et al. (2020).

In the next section, the formal bilateral CVA expression for any OTC derivative, assuming the absence of dependency between its defining components, is first deduced. A bilateral CVA model with dependencies is then proposed, modelling a hazard rate for each entity as a dependent exponential function of the value of the derivative and of its own exposure to risk, in order to capture the WWR and the correlation between the defaults of the two entities. In the third section, the IRS's fair value is deduced by subtracting the proposed bilateral CVA with dependencies from the risk-free value of the swap. It is assumed that the interest rate follows the Vasicek model (1977), while Monte Carlo simulation is used to obtain the different scenarios for the value of the swap and its credit risk exposure at any moment in time. By so doing, the hazard rate for each entity can be specified and their probability of default determined. Finally, the fair value of an IRS is numerically deduced using the proposed bilateral CVA model with dependencies. This result is compared to the outcome obtained when applying the Hull-White model (2012). This process is repeated in ten packages of 10,000 simulations to validate the results obtained. The main conclusions from the study are presented in the last section.

2. Bilateral CVA with WWR and default correlation

The credit risk of OTC derivatives varies over time and is bilateral. This being the case, the counterparty credit risk adjustment on the value of these derivatives has to take into consideration not only the credit risk attributable to the default of counterparty B, but also that of entity A which prices the derivative.

The credit risk adjustment of counterparty B is calculated as the present value of expected losses should this counterparty fail to meet its payment obligations at the scheduled dates. For entity A, the credit risk adjustment is the present value of expected losses due to its own default. This adjustment increases the value of the derivative because of the possibility that, from hereon in, entity A fails to make the expected payments on the derivative (Basel Committee on Banking Supervision, 2015).

The bilateral CVA between entities A and B is obtained from EYG (2014) (see, also Cherubini, 2013):

$$CVA = LGD \int_0^T EE^A(t) f(0,t) p^B(t) dt - LGD \int_0^T EE^B(t) f(0,t) p^A(t) dt$$

which can be approximated by:

$$CVA = LGD \sum_{i=1}^{n} EE_{t_i}^A f(0,t_i) p_{t_i}^B - LGD \sum_{i=1}^{n} EE_{t_i}^B f(0,t_i) p_{t_i}^A \quad [1]$$

where the loss given default (LGD) is the percentage of the exposure that can be expected to be lost in the event of default. Similarly, $LGD = 1 - RR$, where RR is the recovery rate, which is assumed to be constant and the same for both entities.

The asset is considered as having a maturity of T years and t_i, with $i = 1,2,\dots,n$, are the settlement dates of the derivative. As such, the time to maturity is divided in n periods. Moreover, $t_0 = 0$ and $t_n = T$ and it is assumed that the event of default can only occur in t_i.

Additionally, $EE_{t_i}^A$ is the expected exposure of entity A if entity B defaults in t_i, and $EE_{t_i}^B$ is the expected exposure of entity B if entity A defaults in t_i, where the expected exposure is the loss each entity would suffer if its counterparty were to default, which is quantified by the average value of the derivative in t_i. Most studies obtain the value of the derivative, in each t_i, using Monte Carlo simulation.

Assuming there are no collateral arrangements, the credit risk exposure of a derivative in t_i, for entities A and B is, respectively:

$$E_{t_i}^A = Max(+V_{t_i}, 0) \text{ and } E_{t_i}^B = Max(-V_{t_i}, 0) \quad [2]$$

where V_{t_i} is the financial value of the derivative in t_i, i.e., the amount agents would be expected to exchange in the event of contract cancellation in t_i. It is assumed that if $V_{t_i} > 0$, this amount would be collected by entity A, while if $V_{t_i} < 0$ the amount would be collected by entity B.

The corresponding expected exposure is obtained from each entity's risk exposure in relation to that of its counterparty:

$$EE_{t_i}^A = E(E_{t_i}^A) \text{ and } EE_{t_i}^B = E(E_{t_i}^B) \quad [3]$$

The factor $f(0,t_i)$ is the risk-free discount function, in $t_0 = 0$, for t_i years:

$$f(0,t_i) = exp(-R(0,t_i)t_i) \quad [4]$$

where $R(0,t_i)$ is the spot interest rate at $t_0 = 0$, nominal annual with continuous compounding for t_i years.

Finally, $p_{t_i}^B$ y $p_{t_i}^A$ are default probabilities between times t_{i-1} and t_i of entities B and A, respectively (Badía, Galisteo, and Preixens, 2014). They are risk-neutral probabilities, unconditioned to non-default statuses in previous period. These probabilities can be obtained from the corresponding hazard rates. Thus, if $h_{t_i}^B$ is the hazard rate of counterparty B, related to t_i years, the cumulative default probability from $t_0 = 0$ until t_i is $1 - exp(h_{t_i}^B t_i)$; hence, the default probability of B between t_{i-1} and t_i is:

$$p_{t_i}^B = exp(h_{t_{i-1}}^B t_{i-1}) - exp(h_{t_i}^B t_i) \quad [5]$$

The hazard rate, $h_{t_i}^B$, can be approximated by the credit spread for the maturity of t_i years, $s_{t_i}^B$:

$$h_{t_i}^B = \frac{s_{t_i}^B}{1-RR} \quad [6]$$

The same reasoning can be applied to obtain the default probability of entity A for t_i years, where $s_{t_i}^A$ is the credit spread:

$$p_{t_i}^A = exp(h_{t_{i-1}}^A t_{i-1}) - exp(h_{t_i}^A t_i) \text{ with } h_{t_i}^A = \frac{s_{t_i}^A}{1-RR} \quad [7]$$

Here, this paper proposes a model for the bilateral CVA that incorporates the WWR; in other words, it takes into account the positive or negative dependency between default and the value of the financial asset. Furthermore, the model incorporates the dependency between the default of entity A and that of entity B. This modification to the Hull-White proposal (2012) models the hazard rate as an exponential function of the value of the portfolio of derivatives, V_{t_i}:

$$h_{t_i} = exp(a_{t_i} + bV_{t_i} + \sigma\varepsilon)$$

Additionally, to incorporate the correlation between entity A and B's defaults, the model includes an exponential function of the hazard rate, which for entity B is:

$$h_{t_i}^B = exp(a_{t_i} + bV_{t_i} + cE_{t_i}^B + \sigma\varepsilon)$$

WWR can then be introduced by making entity B's hazard rate dependent on the value of the derivative, V_{t_i}, through a parameter b. The dependency between the two counterparties' defaults is introduced by incorporating entity B's exposure to risk, $E_{t_i}^B$, through a parameter c, given that this exposure represents what B is expected to lose in the event of A defaulting. Parameters b and c are assumed to be constant and a_{t_i} is a time-dependent parameter. The disturbance term, ε, is a standardized random variable and σ is assumed to be constant.

According to Hull and White (2012), the hazard rate is not influenced greatly by the inclusion or otherwise of this disturbance term. Ignoring this term, the hazard rate expression of entity B is approximated as:

$$h_{t_i}^B = exp(a_{t_i} + bV_{t_i} + cE_{t_i}^B) \quad [8]$$

Similarly, the hazard rate of entity A is approximated as:

$$h_{t_i}^A = exp\left(a'_{t_i} + b'V_{t_i} + c'E_{t_i}^A\right) \quad [9]$$

Entity A's default is a function of the value of the derivative and of the default of its counterparty B, given that it depends on what A can be expected to lose in the event of B defaulting. Likewise, in this case, parameters b' and c' are constant and a'_{t_i} varies over time.

Following Hull and White (2012), for the hazard functions of entities B and A to be positive, $h_{t_i}^B \geq 0$ and $h_{t_i}^A \geq 0$, and the following condition needs to be verified for both rates:

$$E\left[exp\left(-h_{t_i}^B t_i\right)\right] = exp\left(-\frac{s_{t_i}^B}{1-RR} t_i\right),$$

$$E\left[exp\left(-h_{t_i}^A t_i\right)\right] = exp\left(-\frac{s_{t_i}^A}{1-RR} t_i\right) \quad [10]$$

From hazard rate expressions [8] and [9], the default probabilities for entities B and A, $p_{t_i}^B$ and $p_{t_i}^A$, are obtained, in accordance with [5] and [7].

Thus, to complete the bilateral CVA calculation in line with the methodology proposed in this section, expression [1] has to be applied, with the values of the previous probabilities that incorporate both the WWR and the dependency between the default of the respective entities.

3. Numerical results. Fair value of an IRS

The model outlined above for obtaining the bilateral CVA with dependencies is applied here to determine the credit risk-adjusted value of one of the most actively traded OTC derivatives on the market, namely the IRS.

The fair value of an IRS at $t_0 = 0$, V_0^*, is obtained by subtracting the amount of the bilateral CVA with dependencies from its risk-free value at that time, V_0 (Aragall, 2013):

$$V_0^* = V_0 - CVA \quad [11]$$

At time $t_0 = 0$, the risk-free value (that is, the value assuming there is no default), V_0, of an IRS with nominal N and interest payments that are settled at t_k with $t_k - t_{k-1} = P$ years, $k = 1,2,\dots,n$, and to which n periods are left until the maturity date, is obtained as the difference between the current value of the pending floating interest payments, V_0^v, and the current value of the pending fixed interest payments, V_0^f (Badía, Galisteo, and Preixens, 2011):

$$V_0 = V_0^v - V_0^f \quad [12]$$

The entity valuing the swap, i.e. entity A, is assumed to be the one paying fixed interest payments and the one collecting floating interest payments.

The value at $t_0 = 0$ of the floating leg of IRS, V_0^v, is:

$$V_0^v = \sum_{k=1}^{n} Y_{t_k} f(0, t_k) \quad [13]$$

where the floating payment of the swap corresponding to period k is:

$$Y_{t_k} = N[exp(R(t_{k-1}, t_k)P) - 1] \quad [14]$$

and $f(0, t_k)$ is calculated according to [4].

Additionally, $R(t_{k-1}, t_k)$ is the spot rate at t_{k-1}, nominal annual with continuous compounding, for $t_k - t_{k-1}$ years. At $t_0 = 0$ interest rates $R(t_{k-1}, t_k)$, with $k = 2,3,\dots,n$, are unknown. Thus, to calculate the value of the floating leg, $R(t_{k-1}, t_k)$ can be replaced by the corresponding forward interest rates or an alternative method to that in expression [13] – the zero-coupon method – can be used that allows us to obtain the same result:

$$V_0^v = N - Nf(0, t_n) \quad [15]$$

The value at $t_0 = 0$ of the fixed leg of the swap, V_0^f, is:

$$V_0^f = Y \sum_{k=1}^{n} f(0, t_k) \quad [16]$$

where $Y = N[exp(R^F P) - 1]$ is the fixed interest payment of the IRS for each period, and R^F is the nominal, annual and fixed interest rate with continuous compounding agreed to between the two swap agents on entering into the contract.

To calculate the CVA, the value of the IRS at any t_i, V_{t_i}, has to be determined:

$$V_{t_i} = V_{t_i}^v - V_{t_i}^f = \sum_{k=i}^{n} Y_{t_k} f(t_i, t_k) - Y \sum_{k=i}^{n} f(t_i, t_k) \text{ with } i = 1,2,\dots,n \quad [17]$$

The discount factor between t_i and t_k is:

$$f(t_i, t_k) = exp\left(-R(t_i, t_k)(t_k - t_i)\right) \quad \text{with} \quad i = 1,2,\dots,n \text{ and } k = i, i+1,\dots,n \quad [18]$$

Alternatively, the value of the floating leg of the swap, $V_{t_i}^v$, can be obtained by:

$$V_{t_i}^v = Y_{t_i} + N - Nf(t_i, t_n) \quad [19]$$

Thus, from entity A's point of view (that is, of the fixed-rate payer), a positive V_{t_i} indicates the amount it would no longer collect in t_i in the event of the default of entity B, while a negative value indicates what the same entity (i.e. A) would cease to pay in the event of its going into default.

To determine the CVA, the Monte Carlo method is used (Korn, Korn, and Kroisandt, 2010). This allows M simulations to be obtained, at each t_i, of the spot rate curve.

The evolution of the rate curve is assumed to follow the Vasicek process (1977), a one-factor model of the instantaneous interest rate that follows a stochastic Ornstein-Uhlenbeck process with mean reversion:

$$dr(t) = \alpha\left(\gamma - r(t)\right)dt + \rho\, dz(t),$$

where α is the speed-of-adjustment parameter, γ is the long-term mean of the instantaneous interest rate and ρ is the process volatility.

At $t_0 = 0$ and for each t_i, M instantaneous interest rates are simulated. For every $m = 1,2,\dots,M$ simulations, the spot rate curve is obtained at t_i, $R_m(t_i, t_k)$, $i = 1,2,\dots,n$ and $k = i+1, i+2,\dots,n$, which according to the Vasicek model, with $\tau = t_k - t_i$, is:

$R(\tau, r) = -\frac{\ln A(\tau)}{\tau} + \frac{B(\tau)}{\tau} \cdot r(t),$

where:

$$A(\tau) = exp\left\{-\frac{\rho^2}{4\alpha}B(\tau)^2 + \left(\gamma + \frac{\rho\lambda}{\alpha} - \frac{1}{2}\frac{\rho^2}{\alpha^2}\right)\left(B(\tau)\right) - \tau\right\},$$

$$B(\tau) = \frac{1 - e^{-\alpha\tau}}{\alpha}.$$

Thus, at every t_i and from the simulated interest rates, M simulations of the value of the IRS, V_{m,t_i}, are obtained, which allows us to obtain M simulated values of the exposure to risk of both entities A and B, according to [2]:

$$E^A_{m,t_i} = Max\left(+V_{m,t_i}, 0\right),\ E^B_{m,t_i} = Max\left(-V_{m,t_i}, 0\right) \quad [20]$$

From each entity's exposure to risk in relation to that of its counterparty, we can obtain its corresponding expected exposure, according to [3]:

$$EE^A_{t_i} = E\left(E^A_{m,t_i}\right),\quad EE^B_{t_i} = E\left(E^B_{m,t_i}\right) \quad [21]$$

The expected exposure of the IRS at t_i is the amount that, at this particular moment in time, an entity is expected to lose in the event of the default of its counterparty (Morales, 2014). This expected loss is the average of the simulated values of the swap, immediately before the settlement amount becomes effective. These swap values may be either positive or negative. Thus, the expected exposure for the entity valuing the swap, i.e. A, the fixed-rate payer, can be obtained from the average of the positive values, while the expected exposure of entity B corresponds to the average of the negative values and takes into account the counterparty credit risk of entity A.

According to the model proposed herein, the default probability of each of the entities depends on its corresponding hazard rate, which is expressed as the swap's value functions and each entity's exposure to risk. Functions [8] and [9] can now be approximated, respectively, by means of the simulations thus obtained:

$$h^B_{m,t_i} = exp\left(a_{t_i} + bV_{m,t_i} + cE^B_{m,t_i}\right),$$
$$h^A_{m,t_i} = exp\left(a'_{t_i} + b'V_{m,t_i} + c'E^A_{m,t_i}\right) \quad [22]$$

The fair value of an IRS of nominal 100, $N = 100$, with annual settlement of the payments of interest, $P = 1$, and 5 years until maturity, $n = 5$, is thus obtained. The entity valuing the swap is the party that pays the fixed-rate payments at 4.25% annual nominal interest rate, where $R^F = ln\, 1{,}0425$. The goal now is to compare the fair value of the IRS calculated using the model developed herein with that obtained using the Hull-White model (2012).

First, the current risk-free value of the swap, V_0, is obtained. To this end, and relying on Vasicek's model (1977), the spot interest rates can be determined at $t_0 = 0$ for t_k years, $R(0, t_k)$, $k = 1,2, \ldots, 5$, assuming that the instantaneous interest rate is 3% and that the diffusion parameters that define its stochastic evolution are a speed of mean reversion of 0.015, a long-term mean of 0.25, a risk premium of 0 and a volatility of 0.01. These data are inspired on Hull's work to make a numerical application of the model.

Based on the above data, the yield curve and the discount factors, for $t_0 = 0$, can be deduced (see Table 1).

Table 1: Yield curve and discount factors

k	$R(0, t_k)$%	$f(0, t_k)$
1	3.162530	0.968870
2	3.320206	0.935753
3	3.473154	0.901050
4	3.621496	0.865144
5	3.765353	0.828393

Source: Authors' own

The risk-free value of the swap, according to [12] [15] and [16] is:

$$V_0 = 100[1 - f(0, t_5)] - 4.25 \sum_{k=1}^{5} f(0, t_k) = -1.960932$$

Next, the value of the bilateral CVA is calculated, applying [1]:

$$CVA = LGD \sum_{i=1}^{5} EE^A_{t_i} f(0, t_i) p^B_{t_i} - LGD \sum_{i=1}^{5} EE^B_{t_i} f(0, t_i) p^A_{t_i} \quad [23]$$

A 40% recovery rate is assumed, thus $LGD = 0.60$. The Monte Carlo method is applied to Vasicek's model (1977) to obtain, at $t_0 = 0$, the simulated values of the yield curve $R_m(t_i, t_k)$ with $k = i + 1, i + 2, \ldots, 4$; $i = 1,2, \ldots, 4$ and $m = 1,2, \ldots, 10{,}000$. To validate the results obtained, this process is applied to ten packages of 10,000 simulations each[1].

For each set of 10,000 simulations, by applying [17] and [19], the simulated spot rates allow us to obtain, in turn, 10,000 simulations of the risk-free value of the swap in each t_i:

$$V_{m,t_i} = Y_{m,t_i} + 100 - 100 f_m(t_i, t_5) - 4.25 \sum_{k=i}^{5} f_m(t_i, t_k)$$

where $i = 1,2, \ldots, 5$, and $m = 1,2, \ldots, 10{,}000$, where the simulations of both the floating interest rates and the discount factors in each settlement date are, respectively, according to [14] and [18]:

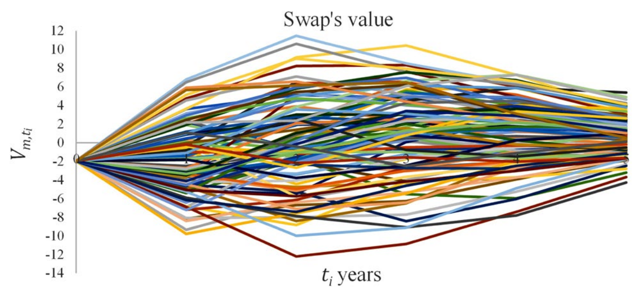
[1] The computation time for the total 100,000 simulations used in this paper (10 sets of 10,000 simulations) takes around 60 seconds on an M1 processor with 8 cores. The average computation time for a single simulation is 0.623 ms. The code to compute the simulations was implemented using the Python programming language.

$Y_{m,t_i} = 100\left[exp\left(R_m(t_{i-1}, t_i)\right) - 1\right]$,

$f_m(t_i, t_k) = exp\left(-R_m(t_i, t_k)(t_k - t_i)\right)$

Figure 1 shows, for the first package, 100 of the 10,000 simulations of V_{m,t_i}, for $i = 1, 2, 3, 4, 5$

Figure 1: Swap value simulations



Source: Authors' own

From the simulated IRS values, each entity's simulations of exposure to risk are obtained, as well as at every t_i, according to [20]

$E^A_{m,t_i} = Max\left(+V_{m,t_i}, 0\right)$, $E^B_{m,t_i} = Max\left(-V_{m,t_i}, 0\right)$, with $i = 1,2, \ldots, 5$

Assuming that all simulated values are equiprobable, each entity's expected exposure, calculated according to [21] and for all ten simulation packages, is as presented in Table 2

Table 2: Expected exposure

Expected exposure A, $EE^A_{t_i}; i = 1, \ldots, 5$					
Package	1	2	3	4	5
1	0.65865238	1.42845097	1.69433236	1.53075165	0.96476382
2	0.65115742	1.43252860	1.70521287	1.53684296	0.96598492
3	0.66138436	1.44713006	1.72084604	1.55563170	0.98119851
4	0.65464464	1.41942062	1.67647354	1.51398263	0.95583567
5	0.65802007	1.44511169	1.70538924	1.53641579	0.96968098
6	0.65389446	1.43391502	1.70141099	1.53498980	0.65389446
7	0.66602981	1.43993235	1.70197450	1.53109927	0.96475336
8	0.65997349	1.43008265	1.69042348	1.52593367	0.96475704
9	0.65704302	1.44268442	1.71103798	1.53988899	0.96819154
10	0.65244318	1.42126467	1.68914791	1.52030738	0.95760923
Expected exposure B, $EE^B_{t_i}; i = 1, \ldots, 5$					
Package	1	2	3	4	5
1	2.68164815	2.40378535	1.91506069	1.31603746	0.66694677
2	2.66887020	2.39463441	1.91259817	1.31224181	0.66312766
3	2.65652650	2.37613070	1.90094492	1.30650556	0.66200027
4	2.67258733	2.39569429	1.91129982	1.31165894	0.66360219
5	2.66472856	2.37793945	1.90170511	1.30589751	0.66091540
6	2.68025482	2.39826259	1.90356595	1.30121355	2.68025482
7	2.67845955	2.40441823	1.91129258	1.30829708	0.66297430
8	2.65588691	2.37361253	1.88315657	1.28906959	0.65437649
9	2.65588691	2.37361253	1.88315657	1.28906959	0.65437649
10	2.67621887	2.40067789	1.90789916	1.30109318	0.65708063

Source: Authors' own

All the results presented up to this point are used to determine the CVA when applying the Hull-White model (2012) and the modified model developed herein, which adds the correlation between the respective defaults of the entities party to the financial derivative. Moreover, the addition of dependencies to these two models affects the value of the default probabilities that are dependent on the specified hazard rate and which differ in each model.

In this section, hazard rates are estimated for both models and both entities (A and B). This process is repeated for all ten simulation packages. In the case of the model proposed herein, these rates are specified in [22] while in the case of Hull and White (2012), they are as follows:

$$h^B_{m,t_i} = exp\left(a_{t_i} + bV_{m,t_i}\right),\ h^A_{m,t_i} = exp\left(a'_{t_i} + b'V_{m,t_i}\right)$$

In line with Hull and White (2012), starting from an initial hypothesis regarding the value of the asset and a 5-year credit spread, the risk exposure and hazard rate are determined for entity A, at $t_0 = 0$, according to [7] and for entity B according to [8]. This information is summarized in Tables 3 and 4.

Table 3: Entity A

V_0	E_0^A	s_0^A %	h_0^A
3	3	3	0.050
-5	0	7.80	0.030
20	20	6	0.100

Source: Authors' own

Table 4: Entity B

V_0	E_0^B	s_0^B %	h_0^B
3	0	3.90	0.065
-5	5	5.40	0.090
20	0	0.72	0.012

Source: Authors' own

From the information above, time independent parameters a_0, b, a'_0 and b' from Hull-White model (2012) can be determined by solving the following equation system:

$$\left.\begin{aligned} exp(a_0 + 3b) = 0.065 \\ exp(a_0 - 5b) = 0.09 \end{aligned}\right\},\ \left.\begin{aligned} exp(a'_0 + 3b') = 0.05 \\ exp(a'_0 - 5b') = 0.03 \end{aligned}\right\}$$

obtaining:

$$a_0 = -2.611334, b = -0.040677$$
$$a'_0 = -3.187291, b' = 0.063853$$

Likewise, the modified model's parameters a_0, b, c and a'_0, b', c' can be calculated by solving the following equation system:

$$\left.\begin{aligned} exp(a_0 + 3b) = 0.065 \\ exp(a_0 + 20b) = 0.012 \\ exp(a_0 - 5b + 5c) = 0.09 \end{aligned}\right\},$$

$$\left.\begin{aligned} exp(a'_0 + 3b' + 3c') = 0.05 \\ exp(a'_0 + 20b' + 20c') = 0.10 \\ exp(a'_0 - 5b') = 0.03 \end{aligned}\right\}$$

obtaining:

$$a_0 = -2.43522,\ b = -0.09938,\ c = -0.09392$$
$$a'_0 = -3.11805,\ b' = 0.07770,\ c' = -0.03692$$

Parameters a_{t_i} and a'_{t_i} are obtained based [10] taking into account the simulated values for the hazard rate:

$$\frac{1}{M}\sum_{m=1}^{M}\left[exp\left(-\sum_{i=1}^{j} h_{m,t_i}\Delta t\right)\right] = exp\left(-\frac{s_j}{1-RR}t_j\right)$$

for $i = 1,2,\ldots,5$ with $1 \le j \le n$, $M = 10{,}000$, $n = 5$, $\Delta t = 1$, $RR = 0.40$. It is assumed that s_j is constant, while for entity A it is 1% and for entity B it is 1.5%.

Applying an iterative process and by means of a bisection method, with 0.000005 tolerance, a_{t_i} and a'_{t_i} values are deduced for both models and for the ten sets of 10,000 simulations. Tables 5 and 6 show, by way of illustration, the estimated hazard functions for the first group of 10,000 simulations:

Table 5: Values of a_{t_i} and a'_{t_i}. Hull-White model

i	$h^B_{m,t_i} = exp\left(a_{t_i} - 0.040677V_{m,t_i}\right)$	$h^A_{m,t_i} = exp\left(a'_{t_i} + 0.06385V_{m,t_i}\right)$
	a_{t_i}	a'_{t_i}
1	- 3.78204346	- 3.99230957
2	- 3.74633789	- 4.07531738
3	- 3.71276855	- 4.11865234
4	- 3.68896484	- 4.13085938
5	- 3.67919922	- 4.11987305

Source: Authors' own

Table 6: Values of a_{t_i} and a'_{t_i}. Modified model

i	$h^B_{m,t_i} = exp\left(a_{t_i} - 0.09938V_{m,t_i} - 0.09392E^A_{m,t_i}\right)$	$h^A_{m,t_i} = exp\left(a'_{t_i} + 0.0777V_{m,t_i} - 0.03692E^A_{m,t_i}\right)$
	a_{t_i}	a'_{t_i}
1	- 3.64746094	- 3.94226074
2	- 3.58459473	- 4.00634766
3	- 3.55834961	- 4.04846291
4	- 3.56323242	- 4.07348633
5	- 3.60290527	- 4.08691406

Source: Authors' own

Substituting the simulated values of the IRS, V_{m,t_i}, and the exposure to risk of each entity, E^B_{m,t_i} and E^A_{m,t_i}, in the previous hazard functions, 10,000 simulations of $h^B_{t_i}$ and $h^A_{t_i}$ are obtained for each of the models considered and for each simulation set.

For every t_i the average hazard rate is calculated and substituted in expressions [5] and [7] of default probabilities.

The average hazard rates obtained for the first simulation group are shown in Tables 7 and 8:

Table 7: $\bar{h}^A_{t_i}$ and $\bar{h}^B_{t_i}$. Hull-White model

i	$\bar{h}^A_{t_i}$	$\bar{h}^B_{t_i}$
1	0.01666936	0.02500933
2	0.01669561	0.02501961
3	0.01671790	0.02505291
4	0.01671573	0.02504161
5	0.01669711	0.02502355

Source: Authors' own

Table 8: $\bar{h}^A_{t_i}$ and $\bar{h}^B_{t_i}$. Modified model

i	$\bar{h}^A_{t_i}$	$\bar{h}^B_{t_i}$
1	0.01667471	0.02500439
2	0.01669186	0.025028079
3	0.01670137	0.025047729
4	0.01670481	0.025053023
5	0.01669371	0.02504490

Source: Authors' own

Tables 9 and 10 show the probabilities of default obtained for entities A and B, in the ten packages of 10,000 simulations, for the Hull-White model and the modified model developed herein, respectively:

Table 9: Probabilities of default. Hull-White model

Values of $p_{t_i}^A$					
Package	1	2	3	4	5
1	0.01653120	0.01630869	0.01607688	0.01575983	0.01541901
2	0.01654016	0.01628257	0.01608581	0.01577616	0.01538966
3	0.01653786	0.01629314	0.01606734	0.01577823	0.01543877
4	0.01653249	0.01631526	0.01603645	0.01580472	0.01542164
5	0.01653608	0.01631195	0.01602866	0.01579323	0.01543777
6	0.01653478	0.01631291	0.01603704	0.01579114	0.01543308
7	0.01653617	0.01630047	0.01606412	0.01577605	0.01544182
8	0.01653543	0.01629738	0.01608393	0.01575129	0.01543714
9	0.01653288	0.01630954	0.01605412	0.01577943	0.01543368
10	0.01653746	0.01630693	0.01603189	0.01579884	0.01542664
Values of $p_{t_i}^B$					
Package	1	2	3	4	5
1	0.02469918	0.02410869	0.02359588	0.02290943	0.02229384
2	0.02469670	0.02412366	0.02354810	0.02296521	0.02228428
3	0.02469242	0.02413755	0.02354469	0.02292904	0.02234631
4	0.02469902	0.02411542	0.02357051	0.02290561	0.02233764
5	0.02469668	0.02411611	0.02356210	0.02296392	0.02228875
6	0.02469267	0.02414015	0.02353710	0.02295218	0.02229669
7	0.02469890	0.02412438	0.02354999	0.02293804	0.02231274
8	0.02469918	0.02412191	0.02354722	0.02295982	0.02228639
9	0.02469369	0.02413425	0.02353557	0.02297100	0.02228104
10	0.02469855	0.02411136	0.02357420	0.02295263	0.02225297

Source: Authors' own

Table 10: Probabilities of default. Modified model

Values of $p_{t_i}^A$					
Package	1	2	3	4	5
1	0.01653645	0.01629618	0.01603697	0.01576614	0.01544425
2	0.01653990	0.01629281	0.01604028	0.01575820	0.01542663
3	0.01653669	0.01630096	0.01601509	0.01578085	0.01543549
4	0.01654011	0.01628694	0.01603199	0.01577622	0.01542972
5	0.01653409	0.01630098	0.01604044	0.01574463	0.01546482
6	0.01653127	0.01630562	0.01605397	0.01574108	0.01541441
7	0.01653862	0.01629620	0.01603438	0.01576312	0.01542353
8	0.01653403	0.01629514	0.01606057	0.01572663	0.01543804
9	0.01654072	0.01628352	0.01604294	0.01576446	0.01546254
10	0.01653649	0.01628694	0.01607298	0.01572409	0.01544006
Values of $p_{t_i}^B$					
Package	1	2	3	4	5
1	0.02469437	0.02412961	0.02356535	0.02296514	0.02234672
2	0.02469394	0.02411363	0.02359161	0.02297414	0.02232117
3	0.02469313	0.02413225	0.02354788	0.02301770	0.02228826
4	0.02469496	0.02411062	0.02360277	0.02294564	0.02234426
5	0.02469505	0.02411383	0.02358389	0.02298700	0.02234999
6	0.02469075	0.02413007	0.02357053	0.02299764	0.02229437
7	0.02469535	0.02412441	0.02355884	0.02299639	0.02234001
8	0.02469655	0.02412096	0.02355688	0.02300309	0.02231809
9	0.02469820	0.02411539	0.02355821	0.02301581	0.02231348
10	0.02469139	0.02413322	0.02354429	0.02300834	0.02232930

Source: Authors' own

From expressions [23] and [11] and the results obtained above, the bilateral CVA value and the fair value of the swap are obtained. The following table shows the values of the CVA and the fair value of the swap calculated for both models and for each of the ten sets of 10,000 simulations.

Table 11: Credit valuation adjustment and fair value

	CVA		V_0^*	
Package	Hull-White	Modified	Hull-White	Modified
1	-0.00100295	-0.00090903	-1.95992864	-1.96002256
2	-0.00052480	-0.00043521	-1.96040679	-1.96049638
3	0.00085446	0.00094105	-1.96178605	-1.96187264
4	-0.00145399	-0.00134792	-1.95947760	-1.95958367
5	0.00014467	0.00023988	-1.96107626	-1.96117147
6	-0.00046074	-0.00036594	-1.96047085	-1.96056565
7	-0.00048857	-0.00037485	-1.96044301	-1.96055674
8	0.00005262	0.00015428	-1.96098421	-1.96108587
9	0.00025276	0.00034640	-1.96118435	-1.96127798
10	-0.00111132	-0.00101143	-1.95982027	-1.95992016

Source: Authors' own

After carrying out the first set of 10,000 simulations, the credit risk adjustment provision was found to be greater when the correlation between the defaults of the two entities was added (columns 2 and 3 of Table 11). To confirm this, ten sets of 10,000 simulations each were carried out and the same behaviour was recorded in each case. Thus, the fair value of the swap obtained from the modified model is always lower than that obtained using the Hull-White model (columns 4 and 5 of Table 11). These results show that adding the expected exposure of the company that values the swap when determining the hazard rate results in an increase in the CVA, because it means taking into account the greater credit risk related to the dependency between the default of the two entities that contract the swap.

4. Conclusions

This paper has developed a counterparty credit risk adjustment model to value OTC financial derivatives. The proposal comprises a bilateral CVA with WWR and dependency between the defaults of the entities involved, based on the Hull-White model (2012) which incorporates the hazard rate as an exponential function dependent on the value of the portfolio.

The model presented here – the primary objective of which is to capture counterparty risk more accurately – proposes a modified hazard rate for each entity that includes the credit risk exposure attributable to the counterparty's risk. By so doing, the dependency between the defaults of the entities party to the derivative is added. The incorporation of this modified hazard rate has been found to increase credit risk provisioning.

When applying the model to obtain the fair value of an IRS, the results demonstrate that the bilateral CVA with WWR increases and, as a consequence, the fair value of the swap decreases when the dependency between the entities' defaults is considered. Here, Monte Carlo simulation has been used to determine the CVA and the fair value of the swap. The relationship between these magnitudes for the two models analyzed was found to be the same for the ten sets of simulations performed.

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